MODELLING SHARIAH RISK IN ISLAMIC FINANCE: A PROBABILITY APPROACH

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ABSTRACT

The validity of a contract determines the Shariah compliance status of an Islamic financial institution’s products and services. The contract is deemed valid and effective provided that all the essential elements and the necessary conditions of the contract are fully satisfied. The purpose of this study is to examine the modelling of Shariah risk in a basis of contract’s validity. In particular, this study attempts to measure Shariah risk with respect to the likelihood of the contract will not comply with the Shariah requirements of a valid contract, i.e. Shariah defaults. The present study is conducted using qualitative content analysis method through literature surveys. At the first part, the study reviews binomial model considering the basic assumptions for independent defaults to measure Shariah defaults. By expanding the underlying assumptions, in the second part, the study develops a model of Shariah risk, taking into account the probability of default of one and/or more essential elements of a valid contract. The application shows that Shariah risk can be modelled using the binomial estimation for predicting probability of default of each element considering that the element is independent default. This study pretends to be a guide to the literature, providing a comprehensive list of references and suggesting different possible extensions for its future development particularly in application of Shariah risk model.

Keywords: Contracts; Shariah risk; Modelling; Islamic finance; Probability approach

1. INTRODUCTION

At a time when interest in Islamic finance is growing, it is important to understand how Shariah-based considerations may impact a business or a transaction. This includes measuring, classifying, and quantifying the sorts of risks associated with Shariah compliance. Emphasizing this importance is the relatively recent emergence of Islamic finance and its continuing efforts to develop industry standards and the best practices of Islamic financial institutions (DeLorenzo, 2007).

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In few years before, Shariah risk has been of note more recently with various well-regarded scholars such as a chairman of the Accounting and Auditing Organisation for Islamic Financial Institutions (AAOIFI) Shariah board, Sheikh Taqi Usmani openly questioning practices of Islamic financial institutions. In the year 2009, it was ended with at least 15 cases of sukuk default. The events included up to 85% of all sukuk were non-compliant and subsequent AAOIFI statement clarifying sukuk structures and their validity; the Malaysian High Court Ruling that *bai’ bithaman ajil* (BBA) was not compliant, and the subsequent Malaysian Supreme Court overturning this decision (Firoozye, 2009). Due to that, it has been described as the “default year” for the sukuk market (Khnifer, 2010). Most probably, Shariah risk is seen to be the risk of least well understood or analysed by Islamic financial institutions on a daily basis.

In dealing with Shariah risk, the contract validity is delineated as part of the important indicators to determine whether a transaction can be considered permissible or impermissible (Rosly, 2010; Dusuki et al, 2012; 2013, Oz et al., 2016). A valid contract is one in which all the essential elements and underlying conditions of the contract are fulfilled (Zuhayli, 1996; 2001; Mansuri, 2006; Ayub, 2007). Failure to comply with these principles of Shariah law not only expose to the risk of breaking the trust and confidence of the investors and depositors, but also eventually invokes Shariah risk. This risk exposure, if not paying adequate attention to the whole process of Shariah compliance, may cause unrecognition of income.

Despite of the loss incurred and the Shariah non-compliance income being considered as indicator for Shariah risk measurement, it may have another significant view that, if it is seen from the Shariah default of the contract (Oz et al., 2016). This is because the loss incurred, loss of revenue, Shariah non-compliance income and impact on reputation are such that of the implication of Shariah default. Even that, Shariah risk will adversely affect Islamic financial institutions’ income only when the contracts are deemed invalid in the court of law, either in a foreclosure or ruling via court declaration (Rosly et al, 2017). Therefore, there is a need to have other key risk indicator for Shariah risk prior to that. In fact, Shariah risk is modelled for the default happening due to distress caused by contract defects and not due to the willingness to make default.

Although extensive academic research has explored Shariah risk (such as DeLorenzo, 2007; Sole, 2007; Balz, 2008; El Tify, 2011; Ginena, 2013; Laldin, 2013; Schmid, 2013; Lahsasna, 2014; Fitwi & Elder, 2015; Oz et al., 2016), the Shariah risk modelling has not been touched in the context of Shariah default except a study done by Oz et al. (2016) who stressed on measuring Shariah non-compliance risk through the Shariah non-compliance income. Against this backdrop, this study focuses on the methodology and approach for dealing with Shariah default that causes invalid contract. Specifically, the paper sets out to provide answers to the following research objectives: (1) to measure Shariah risk with respect to Shariah default, and; (2) to propose Shariah risk model through probability of default approach. It is hoped that, by providing the answers to these objectives, this paper can propose a model for Shariah risk measurement.

Following this brief introduction, the study is organized according to the following structure: the next section discusses the theoretical derivation of Shariah risk and the third section provides an insight into the existing Shariah risk measurement. The section also delineates different view of Shariah risk measurement model through the default in Shariah requirements of Islamic law of contract. The fourth section then elaborates on evidence on Shariah defaults as part Shariah risk. The section also gives new default definition with respect to the Shariah default and explains the
binomial estimation model for independent default circumstances. While the fifth section highlights the underlying assumptions for the probability of default in each contract’s element. Hence, the sixth sections provide equations for Shariah risk model, and the final section concludes the study.

2. THE THEORETICAL DERIVATION OF SHARIAH RISK

The present study emphasizes the identification of Shariah risk from the failure of contract to fulfil the essential elements and conditions of a valid contract as laid down in the Islamic finance theory and Islamic law of contract according to the study of Noor et al. (2017). Based on the theoretical approach to derive Shariah risk, referring to the Exhibit 1, the risk can be analysed based on the contract’s validity. Majority of jurists agreed that the contracts can possibly be valid (sahih) or invalid (ghayr sahih). A valid contract is the contract which satisfies all the essential elements such as form of contract, subject matter, and contracting parties as well as their respective conditions (Zuhayli, 1996; 2001; Mansuri, 2006; Ayub, 2007; Najeeb, 2014). It must also be free from prohibited elements such as riba’ and gharar. For instance, a sale contract is deemed as valid when the offer and acceptance are clear and match to one another; the asset is valuable, its existent is clear at the time of the contract, free from uncertainty, fully owned by the seller, and the parties have legal eligibility to complete the contract. Upon the fulfilment of these requirements, the contract is considered as Shariah compliance.

In contrast, invalid contract occurs when there is one or more violations of essential elements or conditions of the contract such as selling blood, pork, and other prohibited commodities, eventually result to Shariah non-compliance. The analysis implies that any circumstance which may render invalidity of the contract will trigger Shariah risk. More importantly, the paper highlights the implications of invalid contract based on the opinion of Hanafi jurists, in which, Shariah risk may be derived from the void (batil) or voidable (fasid) contracts due to the failure of contract to comply with Shariah contractual obligations. Voiding the underlying contracts will not only result to the exposure of risk of loss by Islamic financial institutions, but also reflected in the Shariah non-compliant income (impure income). Meanwhile, the invalid contracts which fall under voidable (fasid) contract are the contracts that not necessarily void. Rather, they may be defected in certain conditions. If rectified, the contracts may result to Shariah compliance and contributing to pure income. Though, a contract is currently concluded as the only approach to determine the status of income derived from any products and transactions conducted by Islamic financial institutions (Rosly, 2010; Dusuki et al., 2012; 2013; Oz et al., 2016).
Exhibit 1: Shariah non-compliance income due to void contract

Source: Author’s summary

3. AN INSIGHT INTO THE SHARIAH RISK MEASUREMENT

Most of the recent studies measure Shariah risk qualitatively. This means that, Shariah risk without any quantitative evidence, is often measured through various definitions in which it is operationally identified (DeLorenzo, 2007; Sole, 2007; Balz, 2008; El Tibly 2011; Ginena, 2013; Schmid, 2013; Lahsasna, 2014) as well as its risk level which resulting from the contract invalidity (Laldin, 2013). Further, Laldin (2013) concluded Shariah risk measurement through the Hanafi categorisation of invalid contracts such as batil and fasid. As refer to Exhibit 2, the identification of Shariah risk resulting from failure to satisfy the essential elements and conditions of Shariah contract has led to the suggestion of Shariah non-compliance income as a proxy for Shariah risk (Oz et al., 2016). The study involved Shariah non-compliance income data that are obtained from annual reports or financial statements of individual Islamic financial institutions. However, according to Oz et al (2016), the approach used in the study is found to be limited to the following conditions:

(i) The disclosures on Shariah non-compliance income in the financial statements do not include any details on the Shariah risk for the different types of contracts.

(ii) Not all Shariah risk events result in financial loss, because most can be rectified, or their income purified.

From that, Shariah risk is seen to be less closely related to the financial loss of Islamic financial institutions that arise from the nullification of contracts, resulting in an adverse impact on the
Islamic financial institutions’ income. Despite that, the loss incurred by Islamic financial institutions has been identified by Rosly et al. (2017) in Malaysian Islamic banking experience as Shariah risk which is originated from credit risk. It is triggered by default of BBA debt obligations, leading to a court hearing for foreclosure wherein the plaintiff and defendant will put their cases before the judge. The loss may include the costs such as compensation and damages paid to customers, returning profit collected from BBA facilities to customers, cost of court proceedings as well as reputation risk. However, these instances reflect to the implication of Shariah risk in Islamic financial institutions. It does not reflect to the what extent the invalidity of contract due to failure to satisfy the essential requirements and conditions of the Shariah contracts as stipulated in Islamic finance theory and Islamic law of contracts will render Shariah risk. This should be emphasized prior to the implication.

This study deviates the current Shariah risk measurement by looking into the probability of default of contract’s essential elements and conditions. This refers to the derivation of Shariah risk discussed in Noor et al. (2017) where Shariah risk is probability that a contract is invalid due to the failure to satisfy three essential elements as well as fulfilling the relevant conditions of the contract such as the subject matter, contracting parties, and form of contract. This study redefines Shariah risk measurement based on the likelihood of default of the contract to meet its Shariah contract’s essential elements and conditions, and on what will be lost if default occurs. In this context, the contract’s failure to meet Shariah contract’s essential elements and conditions is assumed as the default in Shariah, i.e. Shariah default.

**Exhibit 2: Shariah risk measurement**

![Diagram showing Shariah risk measurement](Source: Author’s summary)
4. SHARIAH DEFAULT AS PART OF SHARIAH RISK

According to Kamarudin et al. (2014), default includes the failure to perform on a futures contract as required by an exchange. This means that defaulting on a futures contract occurs when one party does not fulfil the obligations set forth by the agreement. In the event of a default by the contractual party, the court has rights for the termination of the issuance contract certificate.

4.1. New Default Definition

The operational definition of default in this current study refers to the breach of any Shariah requirements under the original terms of the contract agreement between the Islamic financial institutions and the contract holders. The breaching of Shariah requirements may result to the risk of Shariah non-compliance. In this case, we propose a new way of defining default i.e. Shariah default as part of Shariah risk.

The likelihood of default of the contract will not comply with the Shariah requirements of a valid contract depends on the small or large of the occurrence. This includes the default in either one or more contract’s essential elements such as form of contract, subject matter, and contractual parties. Since the elements fall into the pillars of a valid contract in Islamic law of contract, defaulting in their respective conditions will possibly cause contract invalid.

4.2. The Binomial Model For Independent Defaults

The Shariah default may linked to the specific duration. The time of default is up to when such contract is ruled invalid under the court of law or recognised as nullified by Shariah committee (SC), or Shariah risk is understood within the resolutions, policies and procedures approved by the Islamic financial institution’s Shariah board (Lahsasna, 2014).

Under principle 7.1 of Shariah non-compliance risk, Islamic Financial Services Board (IFSB) highlighted the following clauses:

- IIFS\(^1\) shall ensure that they comply at all times with the Shariah rules and principles as determined by the relevant body in the jurisdiction in which they operate with respect to their products and activities. This means that Shariah compliance considerations are taken into account whenever the IIFS accept deposits and investment funds, provides finance, and carries out investment services for its customers.
- IIFS shall ensure that its contract documentation complies with Shariah rules and principles—with regard to formation, termination, and elements possibly affecting contract performance, such as fraud, misrepresentation, duress, or any other rights and obligations.
- IIFS shall undertake a Shariah compliance review at least annually, performed either by a separate Shariah control department or as part of the existing internal and external audit function by persons having the required knowledge and expertise for the purpose.

\(^1\)Institutions Offering Islamic Financial Services
Due to the time of default, the contract may fall into default or non-default in one or more essential elements. Hence, the binomial model is used in this study to represent the discrete processes of Shariah risk circumstances with the assumptions that only two possible outcomes at a time which is default or non-default. This is the simplest and most common probability model for defaults of contract in a homogeneous segment of a portfolio, in which the defaults are independent across form of contract, subject matter, contracting parties, and default occurs with common probability, \( p \). This is the most widely used specification in practice and may be consistent with Basel II requirements calling for a long-run average default probability (Kiefer, 2011).

As far as the defaults are concerned, the behaviour of \( n \) homogeneous contract can be fully described in terms of \( n \) possible scenarios: one default, two defaults and up to \( n \) defaults. The probability \( P_i \) that scenario \( i \) (\( i \) defaults) could happen can be computed using the so-called binomial formula:

\[
P_i = \frac{n!}{i!(n-i)!} (1-p)^{n-i}
\]

For the same contract, Shariah default may occurs mainly in three ways. Firstly, contract may have defaulted by form of contracts under the condition that the subject matter and contracting parties fulfil all conditions. Secondly, contract may have defaulted by subject matter under the condition that the form of contract and contracting parties fulfil all conditions of a valid contract. Thirdly, contract may have defaulted by contracting parties under the conditions that form of contract and subject matter fulfil the conditions of a valid contract. In addition, Shariah default may also occur due to more than one default of contract’s essential elements at a time. It consists of two or all elements are defaults.

Let form of contract = \( X \), subject matter = \( Y \), and contractual parties = \( Z \) are the elements of Shariah defaults. The main possible occurrence of Shariah defaults for the elements consists of:

\[
Shariah\ default = \begin{bmatrix} X & Y & Z \end{bmatrix}
\]

According to Kiefer (2006), the probability approach to describing and modelling default uncertainty is central to risk management and to the requirements of Basel II. In the case of default modelling, where measuring and controlling risk is the aim, it is widely accepted that the probability approach is the correct approach to default uncertainty. There is no serious argument that the probability approach is wrong or inappropriate for modelling uncertain future defaults as well as other unknowns. Therefore, to measure the uncertainty Shariah default for unknown outcomes, the probability of default is the most appropriate approach to be engaged in the methodology. Applying binomial model for probability of Shariah default, the outcomes for the probability of default = 1 and probability of non-default = 0 for respective element can be presented below:
According to Lawson and Marion (2008), having determined the system to be modelled, there is a need to construct the basic framework of the model. This reflects our beliefs about how the system operates. These beliefs can be stated in the form of underlying assumptions. Future analysis of the system treats these assumptions as being true, but the results of such an analysis are only as valid as the assumptions. Thus, Shariah default assumed that if there is probability of default in any essential elements of valid contract which may result to void and voidable contract. Whereas, Islamic law of contract considered form of contract, subject matter, and contractual parties as being variables for contract essential elements.

If the assumptions are sufficiently precise, they may lead directly to the mathematical equations governing the system. To build mathematical model from the sistem analysis, there are three main steps suggested by Lawson and Marion that should be followed; (1) making assumptions, (2) choosing mathematical equation from literature, and (3) solving equation analytically or numerically. Therefore, from the above Shariah default system outcomes, seven underlying assumptions have been identified for the development of propositions.

5. UNDERLYING ASSUMPTIONS

To model Shariah risk with respect to Shariah default, the underlying assumptions on the defaults probabilities have been considered. Considering that the same contract will has zero Shariah risk with no possible defaults occurs as a basic assumption. The contract is completely Shariah compliance as its essential elements and conditions for a valid contract is fulfilled.

**Basic assumption:** Shariah risk is zero due to no Shariah defaults.

Therefore, the underlying assumptions for probability of Shariah default due to default occurs in either one or more essential elements are:

**Assumption 1:** Assuming that Shariah defaulted due to the form of contract does not fulfil the conditions of valid contract providing that the conditions of subject matter and contractual parties have fulfilled.

In this case, if subject matter and contractual parties have fulfilled Shariah requirements according to Islamic law of contract, the probability of default of the contract is the likelihood that one or more conditions of a valid contract have not being complied by the form of contract and fall into default. Therefore, the proposition is:
Proposition 1: Shariah risk consists of probability of default of contract due to form of contract does not fulfil the conditions of a valid contract.

Assumption 2: Assuming that Shariah defaulted due to the subject matter do not fulfil the conditions of valid contract providing that the conditions of form of contract and contractual parties have fulfilled.

This means that, if the form of contract and contractual parties have fulfilled all Shariah requirements according to Islamic law of contract, the probability of default of the contract is the likelihood that the subject matter have breached the contractual obligations. Therefore, the proposition is:

Proposition 2: Shariah risk consists of probability of default of contract due to subject matter do not fulfil the conditions of a valid contract.

Assumption 3: Assuming that Shariah defaulted due to the contractual parties does not fulfil the conditions of valid contract providing that the conditions of form of contract and subject matter have fulfilled.

Hence, if the form of contract and subject matter have fulfilled all Shariah requirements according to Islamic law of contract, the probability of default of the contract is the likelihood that the contractual parties do not fulfil its conditions and fall into default. Therefore, the proposition is:

Proposition 3: Shariah risk consists of probability of default of contract due to contractual parties do not fulfil the conditions of a valid contract.

Assumption 4: Assuming that Shariah defaulted due to the form of contract and subject matter do not fulfil the conditions of valid contract providing that the conditions of contractual parties have fulfilled.

If the contractual parties have fulfilled all Shariah requirements according to Islamic law of contract, the probability of default of the contract is the likelihood that the form of contract and subject matter do not fulfil their conditions and fall into default. Therefore, the proposition is:

Proposition 4: Shariah risk consists of probability of default of contract due to form of contract and subject matter do not fulfil the conditions of a valid contract.

Assumption 5: Assuming that Shariah defaulted due to the form of contract and contractual parties do not fulfil the conditions of valid contract providing that the conditions of subject matter has fulfilled.

If the subject matter has fulfilled all Shariah requirements according to Islamic law of contract, the probability of default of the contract is the likelihood that the form of contract and contractual parties do not fulfil their conditions and fall into default. Therefore, the proposition is:

Proposition 5: Shariah risk consists of probability of default of contract due to form of contract and contractual parties do not fulfil the conditions of a valid contract.
**Assumption 6:** Assuming that Shariah defaulted due to the subject matter and contractual parties do not fulfill the conditions of valid contract providing that the conditions of form of contract has fulfilled.

If the form of contract has fulfilled all Shariah requirements according to Islamic law of contract, the probability of default of the contract is the likelihood that the subject matter and contractual parties do not fulfill their conditions and fall into default. Therefore, the proposition is:

**Proposition 6:** Shariah risk consists of probability of default of contract due to subject matter and contractual parties do not fulfill the conditions of a valid contract.

**Assumption 7:** Assuming that Shariah defaulted due to all essential elements and conditions i.e. form of contract, subject matter, and contractual parties do not fulfill the conditions of valid contract.

In the event that all elements do not fulfill Shariah requirements according to Islamic law of contract, the probability of default of the contract is the likelihood that all elements do not fulfill their conditions and fall into default. Therefore, the proposition is:

**Proposition 7:** Shariah risk consists of probability of default of contract due to form of contract, subject matter, and contractual parties do not fulfill the conditions of a valid contract.

Therefore, Shariah risk model will be designed based on the above assumptions and propositions as summarized in the Table 5.1 below.

<table>
<thead>
<tr>
<th>Shariah default elements</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form of contract, X</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Subject matter, Y</td>
<td></td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Contracting parties, Z</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

Note: ✓ = probability of default

**6. PROBABILITY OF SHARIAH DEFAULT: THE MODEL**

As being discussed earlier, to model Shariah risk, we have redefined Shariah default as part of Shariah risk in which defaulting of form of contract, subject matter, and/or contracting parties will result contract deems invalid. The propositions for the models have been derived from the underlying assumptions. For independent Shariah defaults, the probability of default event in each contract essential elements do not affect each other due to each element has different conditions. Thus, Shariah risk is modelled considering joint probability of Shariah default in contract essential elements. The basic equation for joint probability is \( P (X \cap Y \cap Z) \) (Dokuchaev, 2015). Hence, the model of basic estimation which introduces all elements of a valid Shariah contract is shown in Equation 1 below:
**Basic model:** Shariah risk with no default occurs.

\[
\begin{align*}
\text{Shariah risk} & = P(X \cap Y \cap Z) \\
SR & = P(X) \cdot P(Y) \cdot P(Z) \\
SR & = X \cdot Y \cdot Z \\
SR_{it} & = (XYZ)_{it}
\end{align*}
\]

*(Equation 1)*

Where:

- \( SR_{it} \) = Shariah risk for default \( i \) at time, \( T \)
- \( X_{it} \) = form of contract \( i \) at time, \( T \)
- \( Y_{it} \) = subject matter \( i \) at time, \( T \)
- \( Z_{it} \) = contractual parties \( i \) at time, \( T \)
- \( i \) = Shariah default
- \( t \) = time of default, \( T \)

**Model 1:** Probability of Shariah default across form of contract, \( X \).

Let us consider a homogeneous Shariah portfolio model with \( n \) defaults, where contract can default at time, \( T \). Let \( X = \{ X_i, i = 1, 2, \ldots, n \} \) be a random variable such that:

\[
X_{it} = \begin{cases} 
1 & \text{if form of contract defaults, } i \text{ at time, } T \\
0 & \text{otherwise}
\end{cases}
\]

Assuming that the random variables \( X_1, X_2, \ldots, X_n \) are all independent with identical distribution. Furthermore, \( P[X_{it} = 1] = p \) so that \( P[X_{it} = 0] = 1 - p = q \). Therefore, for particular \( n \) number of default, the probability of contract default due to defect in form of contract is calculated as below:

\[
\text{Probability of default of form of contract} = \sum_{i=1}^{n} pX_{it}
\]

Taking into account the probability of contract default due to form of contract, \( P[X_{it} = 1] = p \) providing subject matter and contractual parties comply with Shariah requirements, Model 1 is proposed in Equation 2 based on model of basic estimation according to proposition 1.

\[
\begin{align*}
SR_{it} & = pqX_{it} \cdot Y_{it} \cdot Z_{it} \\
& = (pqXYZ)_{it} \\
& = \sum_{i=1}^{n} pX_{it}q(YZ)_{it}
\end{align*}
\]

*(Equation 2)*

**Model 2:** Probability of default of contract across subject matter, \( Y \).
Let us consider a homogeneous Shariah portfolio model with \( n \) defaults, where contract can default at time, \( T \). Let \( Y = \{ Y_i, i = 1, 2, \ldots n \} \) be a random variable such that,

\[
Y_{it} = \begin{cases} 
1 & \text{if subject matter defaults, } i \text{ at time, } T \\
0 & \text{otherwise}
\end{cases}
\]

Assuming that the random variables \( Y_1, Y_2, \ldots, Y_n \) are all independent with identical distribution. Furthermore, \( P [Y_{it} = 1] = p \) so that \( P [Y_{it} = 0] = 1 - p = q \). Therefore, for particular \( n \) number of defaults, the probability of contract default due to defect in subject matter is calculated as below:

\[
Probability \text{ of default of subject matter} = \sum_{i=1}^{n} pY_{it}
\]

Taking into account the probability of contract default due to subject matter, \( P [Y_{it} = 1] = p \) providing the form of contract and contractual parties comply with Shariah, Model 2 is proposed in Equation 3 according to proposition 2, as below.

\[
SR_{it} = X_{it} \cdot pqY_{it} \cdot Z_{it} \\
= (XpqYZ)_{it} \\
= \sum_{i=1}^{n} pY_{it}q(XZ)_{it}
\]

\((Equation \ 3)\)

**Model 3:** Probability of default of contract across contractual parties, \( Z \).

Let us consider a homogeneous Shariah portfolio model with \( n \) defaults, and where contract can default at time, \( T \). Let \( Z = \{ Z_i, i = 1, 2, \ldots n \} \) be a random variable such that,

\[
Z_{it} = \begin{cases} 
1 & \text{if contractual parties defaults, } i \text{ at time, } T \\
0 & \text{otherwise}
\end{cases}
\]

Assuming that the random variables \( Z_1, Z_2, \ldots, Z_n \) are all independent with identical distribution. Furthermore, \( P [Z_{it} = 1] = p \) so that \( P [Z_{it} = 0] = 1 - p = q \). Therefore, for particular \( n \) number of defaults, the probability of contract default due to defect in contractual parties is calculated as below:

\[
Probability \text{ of default of contractual parties} = \sum_{i=1}^{n} pZ_{it}
\]

Taking into account the probability of contract default due to contractual parties, \( P [Z_{it} = 1] = p \) providing the form of contract and subject matter comply with Shariah, Model 3 is proposed in Equation 4 according to proposition 3, as below.
\[ SR_{it} = X_{it} \cdot Y_{it} \cdot p q Z_{it} \]
\[ = (X Y p q Z)_{it} \]
\[ = \sum_{i=1}^{n} p Z_{it} q (X Y)_{it} \]

\textit{(Equation 4)}

**Model 4:** Probability of default of contract across form of contract, \(X\) and subject matter, \(Y\).

Taking into account the probability of contract default due to form of contract and subject matter, \(P \left[ X_{it} = 1 \right] = p\) and \(P \left[ Y_{it} = 1 \right] = p\) providing the contractual parties complies with Shariah, Model 4 is proposed in Equation 5 according to proposition 4, as below.

\[ SR_{it} = \ p q X_{it} \cdot p q Y_{it} \cdot Z_{it} \]
\[ = \ [(p_x q_x X)(p_y q_y Y) Z]_{it} \]
\[ = \sum_{i=1}^{n} [(p_x q_x X)(p_y q_y Y) Z]_{it} \]

\textit{(Equation 5)}

**Model 5:** Probability of default of contract across form of contract, \(X\) and contractual parties, \(Z\).

Taking into account the probability of contract default due to form of contract and contractual parties, \(P \left[ X_{it} = 1 \right] = p\) and \(P \left[ Z_{it} = 1 \right] = p\) providing the subject matter comply with Shariah, Model 5 is proposed in Equation 6 according to proposition 5, as below.

\[ SR_{it} = \ pq X_{it} \cdot Y_{it} \cdot p q Z_{it} \]
\[ = \ [(p_x q_x X)(p z q_z Z) Y]_{it} \]
\[ = \sum_{i=1}^{n} [(p_x q_x X)(p z q_z Z) Y]_{it} \]

\textit{(Equation 6)}

**Model 6:** Probability of default of contract across subject matter, \(Y\) and contractual parties, \(Z\).

Taking into account the probability of contract default due to subject matter and contractual parties are \(P \left[ Y_{it} = 1 \right] = p\) and \(P \left[ Z_{it} = 1 \right] = p\), providing the form of contract comply with Shariah, Model 6 is proposed in Equation 7 according to proposition 6, as below.

\[ SR_{it} = \ X_{it} \cdot p q Y_{it} \cdot p q Z_{it} \]
\[ \begin{align*}
\text{Model 7: Probability of default of contract across form of contract, } X, \text{ subject matter, } Y \text{ and contractual parties, } Z. \\
\text{Taking into account the probability of contract default due to form of contract, subject matter, and contractual parties, are } P [X_{it} = 1] = p, P [Y_{it} = 1] = p \text{ and } P [Z_{it} = 1] = p \text{ respectively, providing none is Shariah compliance. Model 7 is proposed in Equation 8 according to proposition 7, as below.}
\end{align*} \]

\[
\begin{align*}
SR_{it} &= pqX_{it} \cdot pqY_{it} \cdot pqZ_{it} \\
&= [(p_x q_x X)(p_y q_y Y)(p_z q_z Z)]_{it} \\
&= \sum_{i=1}^{n} [(p_x q_x X)(p_y q_y Y)(p_z q_z Z)]_{it} \\
\end{align*}
\]

(Equation 8)

The findings indicate that there is another reasonable measurement for Shariah risk on contract’s validity basis. The models show that, default in one of the Shariah contract’s essential elements may trigger Shariah risk. It implies that, fulfilling all essential elements and conditions of a valid contract is crucial to ensure permissible transaction and for the income recognition as well as in minimizing the risk. Therefore, the models would be part of the additional contribution to the previous studies in the area of Shariah risk such as Ginena (2014), Oz et al. (2016) and Rosly et al (2017). The modelling of Shariah risk in this study contribute as a pioneer of an application Shariah risk model in practical.

7. CONCLUSION

Being a subject that has drawn a significant attention particularly over the past few years, Shariah risk requires its preferable measurement. This is because if non-compliance event occurs, the Islamic financial institutions’ operations and business activities may be considered impermissible and unrecognition of income. Instead of using Shariah non-compliance income as a proxy for Shariah risk, this study examines the default in complying Shariah requirements of a valid contract. Therefore, this study has applied binomial estimation model to estimate the probability of Shariah default for each essential elements of contract to measure Shariah risk. Although this simple model is probably the widest use in practice, it is rarely used in a risk modelling especially in the Shariah risk context.
There are seven models have been developed based on underlying assumptions of Shariah defaults. The results suggest that, it is possible for Shariah risk to be modelled using the binomial estimation for predicting probability of default of each element considering that element is independent default. Noting that, these models are applicable for the same type of contract. Refinements of the model are possible especially in development of a single model for Shariah risk. The models also suggest another significant approach to explore Shariah risk implications on the capital adequacy of Islamic financial institutions. Besides, studies also can be done by focusing on the variable testing for a contract using Shariah risk model in Islamic financial institutions. These are certainly topics for future research.

REFERENCES


