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OPTIMAL ASSETS ALLOCATION FOR RISK AVERSE INVESTOR UNDER MARKET RISKS AND CREDIT RISK

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ABSTRACT

This research will study a reduced form model for optimal investment with a defaultable corporate bond under the market risks (deterministic rate of return and inflation risk), and credit risk. Those aforementioned risks are influenced by macro-economic, and considered as an exogenous risk. For inflation rate and credit spread rate will be modelled under the Vasicek model. By using Vasicek model, the mean reverting behaviour of the rates will be reached, since this model tends to have a constant mean in long term period. The data will be taken from the Indonesian rate of inflation (from January 2010 to December 2015). Further, this calculation will be solved using Stochastic Dynamic Programming. The closed form solution will give the proportion of wealth between bond and money account. Furthermore, the composition of the portfolio will be given as the result. The complicated equation of bond pricing will follow recovery market value (RMV) methods. Last the simulated data will be given to validate and calibrate the finding model.

***Keywords**:* Asset Allocation; Optimal Portfolio; Dynamic Statistic Programming; Credit Spread Rate; Vasicek Model.

1. INTRODUCTION

This paper will discuss the problem of an investor who wants to allocate her assets into an optimal portfolio to get an optimal return for her benefit. Beside for having some return, an investor also has to face some risks. The risks that the investor will get, are coming from the market and those are rate of return and rate of inflation. After Asian Crisis in 1997-1998, the central bank of Indonesia has changed its monetary policy framework from having a multiple ultimate targets as for inflation, economy growth and job creation, into a single ultimate target which is inflation rate. It means that the central Bank of Indonesia has mentioned explicitly that they will always interfere the inflation as the last ultimate target for Indonesian Monetary Policy. To reach this target, central bank will set a short term operational targets that will be adapted to the performance of economy and financial markets (Sitorus, 2015). This will follow that the policy of inflation regulation will affect the value of assets.

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In emerging market it is known that since global crisis in 2008, the growth of investment in developing countries has been increased more than developed countries. According to Forbes.com (http://www.forbes.com) the emerging economies have been increased two or three times more than developed countries which makes developed countries such as USA have more interested to invest in developing countries as well.

As a consequence for a growing investment, the investment risks will also increase. In the world of investment, the market risks are not the only risks that the investor will face, there is also the possibility that the obligor will face bankruptcy (default). Default risk becomes more important point of view for corporate bond, especially after global crisis in 2008 which has made United States and the world collapsed. Since then there are many literatures studies that brought more deeply about the default risk issue into their studies. Default risk is usually connected with the credit risk, and it is very fundamental, Hou & Jin (2002). In Indonesia the investment in corporate bond has not yet been widely attractive as in other countries. But according to the report of Asian Development Bank (ADB) in June 2013, the investment in corporate bond has increased and reached for U.S$ 20 billion at the end of March 2013. The growth from 2012 is more than 26% (http://investasi.kontan.co.id, retrieved February 5th, 2014). This makes the issuance of corporate bond will be more interesting compared to loan from the bank as a source of funding. These reasons will bring increasingly good perspective for the corporate bond growth in Indonesia. Therefore the study of modelling the optimal portfolio under market and default risk is interesting to do, especially in Indonesia.

Furthermore there is a big gap in application of portfolio choice between the practical (industry) point of view and the academic point of view. In practical approach, the static framework is often used. Static here means that that the portfolio was replicated for a given static assets, it does not capture the problem of the investor, while in the practice the conditions as well as personal preferences will change constantly. Different with dynamic portfolio, which means that the portfolio is replicated for a given assets and for a small change of underlying parameters, e.g. time, the price of assets will vary and adjust continually with the portfolio itself.

2. LITERATURE REVIEW

The study in dynamic portfolio itself has been widely done by other researchers, but it is rarely connecting the rate of return, rate of inflation and credit spread all together into the asset pricing, where those are usually connected into bond pricing. Within the dynamic portfolio model, we have to define first the fundamental model of portfolio. The choices are between continuous-time model and discrete-time model. In continuous-time model, according to Merton (1978), the underlying stochastic variables follow diffusion type motion within continuous sample path, and the trading takes place continuously in time. The solutions will be both simpler and richer than that from the usual discrete-time model assumption. In continuous-time model it was pioneered by Merton (1969,1971), then followed by: Cox & Huang (1989), Karatzas et.al (1991), Zariphopoulou (2001), Brennan & Xia (2002), Hou & Jin (2002), Hou (2003), Zhou & Li (2000), Castaneda-Leyva & Hernandez (2005), Bielecki & Jang (2006), Bo et. al (2010), Jiao & Pham (2011), Jiao et.al (2013), Bo et. al (2013a, b) and any other references in it.

Most of the studies in dynamic portfolio use only deterministic rate of return as market risk, rarely put rate of inflation in their study such as Samuelson (1969), Merton (1969,1971), Campbell et.al (2001), Zariphopoulou (2001), Hou (2003), Hou & Jin (2002), Castaneda-Leyva & Hernandez

(2005), Stoikov & Zariphopolou (2005), Bielecki & Jang (2006), Dai et.al (2009), Callegaro et.al (2012). The study that connecting market risks between stochastic rate of inflation and stochastic rate of return is done by Brennan and Xia (2002), while the studies that connecting market risk and default risk, under rate of return and credit spread are done by Hou (2003) and Hou and Jin (2002) where the differences between both studies are the rate of return, one is using deterministic rate of return, while other using stochastic rate of return.

Since the global crisis in 2008 there are some studies that using credit risk, but again there is no rate of inflation involve. Those studies also use jump diffusion process as to describe the credit risk. Hou (2003) uses time variant of interest rate and credit spread rate while Hou & Jin (2002) uses a deterministic interest rate and time variant function of credit spread rate. Menoncin & Vigna (2013) is studying the mean-variance optimization problem using corporate bond under interest rate. For the last four articles, they consider risks as the exogenous risk, and they used Vasicek model as the model of the risks. For studies in endogenous risks approach; Bo, Wang & Yang (2010) and Bo et. al (2013a, b) used the constant rate of return as the representation of the market risk, and using the credit risk in the form of jump process, both also using infinite horizon time for the objective goal of investment. The differences laid on the utility functions that they used. Ankirchner et.al (2010), Jiao & Pham (2011), Jiao et.al (2013) are using jump diffusion process to describe the credit spread rate and using mean variance approach rather than using utility function as the object of the investment.

3. RESEARCH OBJECTIVES

The purpose of this study is to solve the optimal portfolio problem under interest rate, rate of inflation and credit spread rate. The model will be given in a closed loop formula that will describe the optimal wealth of the portfolio and also the optimal composition of the assets. The assets that will be used in this study are defaultable zero-coupon bond and money account, since the investor is assumed as a risk averse person. For the model of risks we use stochastic model of Vasicek as an exogenous process. This model has been used by Brennan & Xia (2002), Hou (2003) and Hou & Jin (2002) to describe the risks model. The objective is to find the optimal portfolio strategy based on indirect utility function, following Merton (1971). For the sake of clarity and simplicity the rate of return for both bond pricing and money account are considered deterministic. On the other hand, this study will also give how the correlation between the risks will affect the wealth and also the composition of the assets.

Based on those aforementioned reason, this research will investigate on how the market risk and credit risk are linked to the defaultable zero coupon bond pricing model, also to find the closed loop model of portfolio for two assets which are defaultable zero coupon bond and money account. Finally the optimal composition for this model will be obtained.

4. METHODOLOGY

The methodology of this study is the quantitative finance or financial mathematics approach. This approach is rather different with the methodology in management or business field, which used *empirical data* to define the model or building theory (Sekaran and Bougie (2013), Saunders & Lewis (2014)). Financial mathematics approach uses mathematical tool to draw the model from economic theory, stochastic processes, statistics and probability theory. This approach also used mathematic building model to describe the intuition that will be happened in the real world (Steland (2012)), where the starting point of this model will come from the state of the art in the previous studies. Therefore in this study this approach will be used. Further, the calibration will be made using the previous or simulated data to validate or describe the result and using intuition and comparing with the previous model in the previous study, where also the management finance issues will be discussed also further. The research position in this study will combine the market risks as for the rate of inflation, and the credit risk as for the credit spread function, into the asset pricing models. These risks model will be in the form of vasicek model and will be integrated into the asset pricing. In fact the stylized facts of the assets and also the risks that are modeled in vasicek are considered in normal distribution, therefore the use of random walk model is presence in this study. By using random walk, the assets and also the risks have expected mean and median value closed to zero. Although it is known that in reality the distribution style might be more complex than normal distribution but for the sake of simplicity, without loss of generality we assume here that the model of risks is normal distribution.

Furthermore the objective is to find the optimal portfolio where the variable controls, weight of assets, are to steer the optimal portfolio. The assets that we use here are three assets, defaultable coupon bond, equity asset and money account.

5. MARKET RISKS AND CREDIT RISK MODEL

Assuming that both rate of inflation and credit spread rate are following the Ornstein Uhlenbeck process, such that respectively the model for both will be described as follows

$$dI = \kappa_I(\theta_I - I)dt - \sigma_I dW_I \tag{1}$$

where I is the rate of inflation, θ_I is the mean rate of inflation in long term, K_I is the coefficient of the reversion speed of inflation rate towards its long term mean θ_I, σ_I is the volatility coefficient, t is the time and W_I is the Brownian in space of inflation rate.
For the credit spread rate it will be:

$$d\delta = \kappa_\delta(\theta_\delta - \delta)dt - \sigma_\delta dW_\delta \tag{2}$$

where δ is the credit spread rate, θ_δ is the mean credit spread rate in long term, K_δ is the coefficient of the reversion speed of inflation rate towards its long term mean θ_δ, σ_I is the volatility coefficient, σ_δ is the coefficient of credit spread volatility, and W_δ is the Brownian in space of G.

Both rates are under measure of Q as the risk neutral probability measure. Since we have two brownians in different space of measure, dynamic I and δ both may correlate within $dW_I \, .dW_\delta = \rho_{\delta I}\, dt$, with deterministic coefficient of $\rho_{\delta I}$. We need to transform into the physical measure of P since the investor is a risk averse and use the utility under the physical measure. Using Girsanov theorem we adjust the drift under the risk-neutral probability measure Q to the physical probability measure P, thus equation (1) and (2) become:

$$dI = \left[\kappa_I(\theta_I - I(t))dt + \sigma_I(\rho_{\delta I}\bar{\lambda}_\delta + \bar{\lambda}_I)\right]dt + \sigma_I dW_I^P \tag{3}$$

$$d\delta = \left[\kappa_\delta(\theta_\delta - \delta(t))dt + \sigma_\delta(\bar{\lambda}_\delta + \rho_{\delta I}\bar{\lambda}_I)\right]dt + \sigma_\delta dW_\delta^P \tag{4}$$

6. ASSETS PRICING MODEL

The asset model that usually used in the dynamic portfolio studies are categorized within two types, defaultable asset and default free asset. Stock is considered as defaultable asset while bond is usually considered as the risk free asset in the form of government bond. According to Merton (1969) the price process of risk-free asset *P(t)* is defined in equation (5), and this further will be defined as the model of money account.

$$\frac{dP}{P} = r(t)dt \tag{5}$$

where *r* is the rate of return and considered as a deterministic function.

The rate of return is considered as one of the market risks. This is rather different when we talk about the corporate bond, which is affected both rate of return and rate of inflation (Brennan & Xia, 2002), and credit risk (Bielecki & Kurtowski, 2002). In other words, corporate bond is no longer a risk free asset, but it is categorized as defaultable asset.

To model this asset pricing especially when it is linked with the credit risk, there are two methods that can be used to describe it, namely the reduced form method and the structural method. The first method is usually used when it involves the exogenous factor such as market risks and credit risk and the second method is suitable only for the movement of the firm's value. The first method is more applicable because the assets price can be linked with the credit risk (Bielecki & Kurtowski, (2002)).

We define the bond pricing as a defaultable zero-coupon bond, for its clarity intuition and implication. In equation (2) the cash flow process for defaultable zero coupon bond are separated into two situations, on the first term is the value of bond when there is no default happened, the value of bond will be its par value, and the last term is when default happened, the value of bond will be the recovery value upon default. This definition was mentioned by Bielecki & Kurtowski (2002).

$$\begin{aligned} B^d(t,T) &= 1_{t>T} \times P + 1_{t>T} \times R(\tau) \\ &= 1_{t>T} \times P + \int_0^{\tau} R(s)dH \end{aligned} \tag{6}$$

where the time maturity of bond is defined as *T*, the par value is *P*, default time is τ with $\tau \in (T, \infty]$, the recovery value when default occurred is defined as $R(\tau)$, it is recovered in fulfillment of the corporate debt obligation and will be paid only with a fraction of the promised amount upon default. R is also $\mathcal{F}_{\tau-}$measurable, which means that the payment can be made upon available information.

Furthermore, the model of bond pricing is defined under risk neutral probability measure *Q* following definition from Duffie & Singleton (1999). Follows that by their proof the bond pricing under the recovery market value (RMV) will give the neat result such as:

$$B(t,T) = 1_{t>T} \times P \times \mathbb{E}^Q \left(-\int_t^T e^{(r(s)+\lambda(s))} ds | \mathcal{F}_t \right) \tag{7}$$

Within the assumption that all the stochastic function is normally distributed, the bond pricing can be following affine term structures, then eq. (3) becomes:

$$B(t,T) = 1_{t>T} \times P \times \left(\mathbb{E}^Q\left(-\int_t^T e^{(r(s)+\lambda(s))}ds|\mathcal{F}_t\right)\right) + \tfrac{1}{2}Var^Q\left(-\int_t^T e^{(r(s)+\lambda(s))}ds|\mathcal{F}_t\right) \quad (8)$$

Using Ito calculus the first derivation of equation (3) can be found, in the form of

$$\frac{dB(t,T)}{B(t,T)} \quad (9)$$

With some rigorous calculation of the dynamic asset pricing of the corporate bond, which will be linkage with the market risk and credit risk resp. equation (3) and (4), the bond pricing model can be described as follows, (see appendix for its derivation)

$$\frac{dB}{B} = \left[r + I(t) - \eta(t) + \zeta_\delta(t,T)\sigma_\delta\bar{\lambda}_\delta + \zeta_I(t,T)\sigma_I\bar{\lambda}_I\right]dt + \zeta_\delta(t,T)\sigma_\delta dW_\delta^P + \zeta_I(t,T)\sigma_I dW_I^P \quad (10)$$

where:

$$\zeta_I(t,T) = \frac{\exp(\kappa_I(T-t)) - 1}{\kappa_I}$$

$$\zeta_\delta(t,T) = \frac{\exp(\kappa_\delta(T-t)) - 1}{\kappa_\delta}$$

$$\eta = \frac{1-\omega}{\omega}\delta$$

7. OPTIMAL PORTFOLIO MODEL

Define the wealth process of X which is generated by probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where the system had filtration of F. The control process of investor is π assumed to be F adapted to X, whose element are the fraction of assets against the wealth each time $t \in [0,T]$, for maximizing the investor expected utility of terminal wealth. The percentage of wealth in money account will be set as $(1-\pi)$. The wealth process can be written as follows:

$$\frac{dX}{X} = \frac{dB}{B}\pi + \frac{dX}{X}(1-\pi)$$

$$= \{r + \pi[I(t) - \eta(t) + A_1(t)]\}dt + \pi\zeta_\delta(t,T)\sigma_\delta dW_\delta^P + \pi\zeta_I(t,T)\sigma_I dW_I^P \quad (11)$$

where

$$A_1 = \zeta_\delta(t,T)\sigma_\delta\bar{\lambda}_\delta + \zeta_I(t,T)\sigma_I\bar{\lambda}_I$$

Furthermore setting utility function $U(t)$ to find the solution with object is to maximize the utility function, using Hamilton Jacobian and Bellman process under Stochastic Dynamic Programming process, to find the weight of assets. Following Merton (1971), Hou & Jin (2002) we define the indirect utility function as:

$$\mathcal{J}(X,t,\eta,I) = \max_{\pi(s)\in\mathcal{A},t\le s\le T} \mathbb{E}U(X^{\pi}(T)|\mathcal{F}_t) \tag{12}$$

According to Korn and Kraft (2003) that the derivation of bond pricing model must require a Lipschitz condition, which is not applicable to the situation for where the wealth process and the rates are unbounded process. By than the verification theorem of Korn and Kraft (2003) can allow the HJB equation. In this case the state process will be $Y(t,X,\eta,I,\pi) = (X(t,\eta,I,\pi) \quad \eta(t) \quad I(t)\,)'$. Since we have two stochastic processes credit spread rate and inflation rate, the stochastic differential equation (SDE) of the portfolio state process will be:

$$d\mathrm{Y} = \Pi^{Y}\big(t,Y(t)\big)dt + \Sigma^{Y}\big(t,Y(t)\big)dW^{P} \tag{13}$$

where:

$$\Pi(\mathrm{t},\mathrm{Y(t)},\pi) = \begin{pmatrix} X\{r+\pi[I(t)-\eta(t)+A_1(t)]\} \\ \kappa_\delta\big(\theta_\eta-\eta(t)\big)+A_\eta \\ \kappa_I(\theta_I-I(t))+A_I \end{pmatrix} \tag{14}$$

$$\Sigma^{Y}(t,Y(t),\pi) = \begin{pmatrix} X\pi\zeta_\delta\sigma_\delta & X\pi\zeta_I\sigma_I \\ \sigma_\delta & 0 \\ 0 & \sigma_I \end{pmatrix} \tag{15}$$

$$dW^{P} = \begin{pmatrix} dW_\delta^{P} \\ dW_I^{P} \end{pmatrix} \tag{16}$$

The Hamilton Jacobian Bellman equation for the indirect utility function will be

$$\begin{aligned} D^{\pi}\mathcal{J}(X,t,\eta,I) &= \Pi(\mathrm{t},\mathrm{Y(t)},\pi)dt + \tfrac{1}{2}tr\big(\Sigma(\mathrm{t},\mathrm{Y(t)},\pi)'.\Xi.\Sigma(\mathrm{t},\mathrm{Y(t)},\pi)\big)dW^{P} \\ &= J_t + J_x dx + J_\eta d\eta + J_I dI + \tfrac{1}{2}J_{xx}dx^2 + \tfrac{1}{2}J_{\eta\eta}d\eta^2 + \tfrac{1}{2}J_{II}dI^2 + \\ &\quad \tfrac{1}{2}.2\,J_{x\eta}dxd\eta + \tfrac{1}{2}.2\,J_{xI}dxdI + \tfrac{1}{2}.2\,J_{\eta I}d\eta dI \end{aligned} \tag{17}$$

where $\Xi = (1 \quad \rho_{\delta I};\ \rho_{\delta I} \quad 1)$, is the correlation matrix between I and δ, and Jt, J_Y, J_{YY} are the partial derivatives with respect to the variables.

The calculation for $(\Sigma\Xi\Sigma')$ of the state process will be:

$$(\Sigma.\Xi.\Sigma') = \begin{pmatrix} X\pi\zeta_\delta\sigma_\delta & X\pi\zeta_I\sigma_I \\ \sigma_\delta & 0 \\ 0 & \sigma_I \end{pmatrix}\begin{pmatrix} 1 & \rho_{\delta I} \\ \rho_{\delta I} & 1 \end{pmatrix}\begin{pmatrix} X\pi\zeta_\delta\sigma_\delta & \sigma_\delta & 0 \\ X\pi\zeta_I\sigma_I & 0 & \sigma_I \end{pmatrix} = \begin{pmatrix} M_1 & M_2 & M_3 \\ M_2 & M_4 & M_5 \\ M_3 & M_5 & M_6 \end{pmatrix} \tag{18}$$

where

$M_1 = X^2\pi^2[(\zeta_\delta\sigma_\delta)^2 + 2\rho\zeta_\delta\zeta_I\sigma_\delta\sigma_I + (\zeta_I\sigma_I)^2]$
$M_2 = X\pi\sigma_\delta(\zeta_\delta\sigma_\delta + \rho\zeta_I\sigma_I)$
$M_3 = X\pi\sigma_I(\rho\zeta_\delta\sigma_\delta + \zeta_I\sigma_I)$
$M_4 = \sigma_\delta^2$

$M_5 = \rho\sigma_\delta\sigma_I$
$M_6 = \sigma_I^2$

Following that equation (17) becomes:

$$\begin{aligned} D^\pi \mathcal{J}(X,\eta,t,I) = {} & J_t + J_x X(r + \pi[I(t) - \eta(t) + A_1(t)]) + J_\eta\big(\kappa_\delta\big(\theta_\eta - \eta(t)\big) + A_\eta\big) \\ & + J_I(\kappa_I(\theta_I - I(t)) + A_I) + \tfrac{1}{2}J_{xx}(X^2\pi^2[(\zeta_\delta\sigma_\delta)^2 + 2\rho\zeta_\delta\zeta_I\sigma_\delta\sigma_I + (\zeta_I\sigma_I)^2]) \\ & + \tfrac{1}{2}J_{\eta\eta}\sigma_\delta{}^2 + \tfrac{1}{2}J_{II}\sigma_I{}^2 + \tfrac{1}{2}.2\, J_{x\eta}(X\pi\sigma_\delta(\zeta_\delta\sigma_\delta + \rho\zeta_I\sigma_I)) \\ & + \tfrac{1}{2}.2\, J_{xI}(X\pi\sigma_I(\rho\zeta_\delta\sigma_\delta + \zeta_I\sigma_I)) + \tfrac{1}{2}.2\, J_{\eta I}(\rho\sigma_\delta\sigma_I) \end{aligned} \tag{19}$$

By guessing the ansatz function for indirect utility function (equation (12)) we can solve the optimal portfolio problem and also solve the weight of assets. Next theorems are the results after substituting equation (20) into equation (19). With some rigorous calculation then those theorems below can be found.

Theorem 1.

$$\mathcal{J}(t,x,\eta,I) = \begin{cases} g(t).\exp\big[k(t).\eta(t) + \frac{1}{2}l(t)\eta(t)^2 + m(t)I(t) + \frac{1}{2}v(t)I(t)^2\big] & if\ \ \gamma \neq 1 \\ \ln X & if\ \gamma = 1 \end{cases} \tag{20}$$

where g*(t), k(t), l(t), m(t)* and *v(t)* are deterministic function, given that :

$$g(t) = \exp\left(\int_t^T C_0 dt\right) \tag{21}$$

with

$$\begin{aligned} C_0 = {} & -\left(\frac{1-\gamma}{2\gamma}\right)\frac{A_1^2}{A_4} - \big(A_2 k(t) + A_3 m(t)\big) - \left(\frac{1-\gamma}{2\gamma}\right)\frac{A_7^2\, k(t)^2 + A_8^2\, m(t)^2}{A_4} - A_5\big(k(t)^2 + l(t)\big) - \\ & A_6\big(m(t)^2 + v(t)\big) - \frac{(1-\gamma)A_7A_8 - \gamma A_4 A_9}{\gamma A_4}\ k(t)m(t) - \frac{1-\gamma}{\gamma A_4}\big(A_7 k(t) + A_8 m(t)\big) - (1-\gamma)r \end{aligned} \tag{22}$$

$$l(t) = C_3 + \frac{\bar{A}_1}{\bar{A}_{2.}\exp\big(\bar{A}_1(t-T)\big) - C_2} \tag{23}$$

$$v(t) = C_{13} + \frac{\bar{B}_1}{\bar{B}_{2.}\exp\big(\bar{B}_1(t-T)\big) - C_{12}} \tag{24}$$

The boundary between *l(t)* and *v(t)* will be derived as follows:

$$v(t) = \frac{A_7 l(t) - 1}{C_2 l(t) - A_8} \tag{25}$$

For *k(t)* and *m(t)* can be solved in numeric way, given that the relation between those two function will be as

$$\begin{bmatrix} k'(t) \\ m'(t) \end{bmatrix} = \begin{bmatrix} -(C_{22} + C_{23.}l(t)) & -(C_{24} + C_{25.}l(t)) \\ -(C_{28} + C_{29.}v(t)) & -(C_{30} + C_{31.}v(t)) \end{bmatrix} \begin{bmatrix} k(t) \\ m(t) \end{bmatrix} - \begin{bmatrix} -(C_{26}.\, l(t) + C_{27}) \\ -(C_{32}v(t) + C_{33}) \end{bmatrix} \tag{26}$$

This gives the closed loop model of wealth process for this optimal portfolio problem under market risk and credit risk, which can be seen that it is depend on time, and also the risks.

Theorem 2.

The optimal portfolio weight both for bond composition and money account composition can be described as follows:

1. Weight of bond:

$$\pi_B(t) = \frac{1}{\gamma A_4}(I(t) - \eta(t) + A_1) + A_7\big(k(t) + l(t)\eta(t)\big) + A_8(m(t) + v(t)I(t)) \tag{27}$$

2. Weight of money account

$$\pi_P(t) = 1 - \pi(t) \tag{28}$$

This theorem gives insight that the optimal weight of portfolio problem clearly present that there are links between the portfolio model with market and credit risks. Those are dependent to market and credit risks, and also time horizon. Numerical exercises can give more deeply insight of this model. It can also be seen that the weight assets connected to these risks too.

8. DATA SIMULATION AND ANALYSIS

Here in this section the theoretical results will be illustrated. The inflation rate data is taken from Indonesian data (http://www.tradingeconomics.com/indonesia) from January 2010 to December 2015 (see figure (1)). The data is given in monthly data, therefore the data will be taken as long as possible to have the best information from those values. The other data such as bond pricing and credit spread data will be used the simulating data (where the adjustment for credit spread rate data is taken from Hou (2003) and Hou and Jin (2005)), since we want to simulate from several aspects regarding the choice of the investor appetite. Also by using simulated dat,a a better insight from the result can be seen.

The initial values will be given in table 1. The parameters values in that table below were found using Ait-Sahalia model. The data of rate of inflation is processed using Ait-Sahalia tool (http://www.princeton.edu/~yacine/ closedformmle.htm), where in this tool the behavior of maximum likelihood for the data can be taking care of and it can process the values for every parameter that is need such as volatility (σ), Ornstein Uhlenbeck coefficient (κ), and long term run value (θ).

This simulation is done by assuming that the trading is exercised monthly, with time maturity bond is 1 year, the model from theorem 1 and 2 can be simulated and shown from figure (1) to figure (5).

From figure (2) we can see that the utility function goes nice and smoothly following exponential behavior. The parameters that form theorem 1 such as *g(t), k(t), m(t)* and *v(t)* in figure (3) also can be seen running nice and smoothly as well.

Figure 1: Indonesian rate of inflation from January 2010 to December 2015

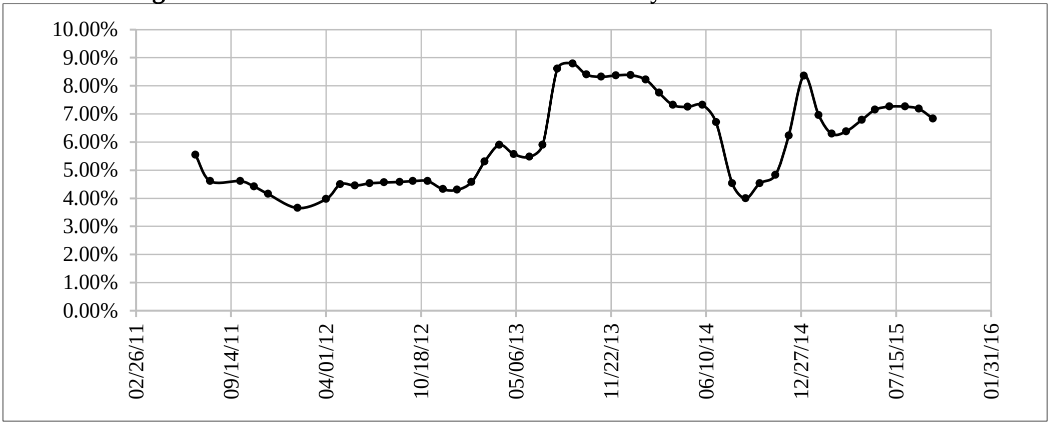

Table 1: Initial ConstantParameters

Symbol	Definition	Values
X	investment value	1
κ_I	Ornstein Uhlenbeck coefficient for inflation rate	0.0545
κ_δ	Ornstein Uhlenbeck coefficient for credit spread rate	0.027
I_o	initial rate of inflation	8%
δ_o	initial value of credit spread rate	30 bps
θ_I	Long term run value for rate of inflation	0.038
θ_δ	Long term run value for credit spread rate	0.012
σ_I	Volatility of rate of inflation	0.014
σ_δ	Volatility of credit spread rate	0.077
ω	Write down rate	0.56
γ	Risk of aversion coefficient	0.6

Figure 2: Plot of utility function

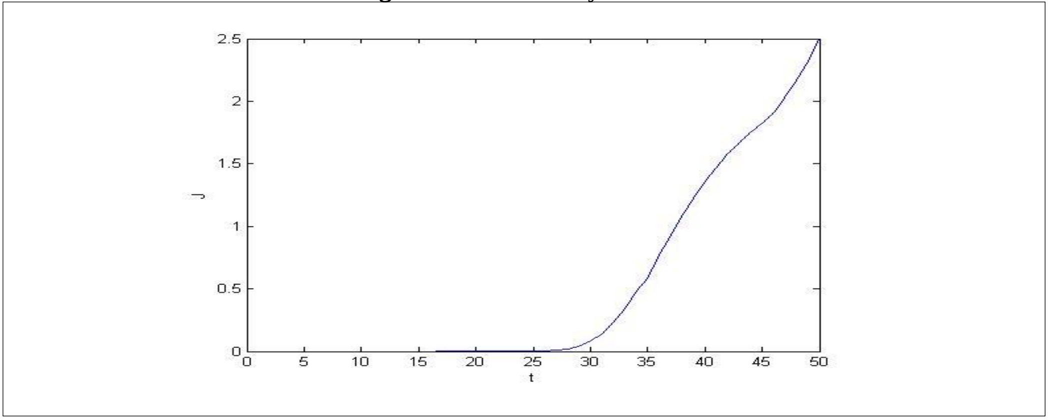

Figure 3: Plot of parameter in utility function

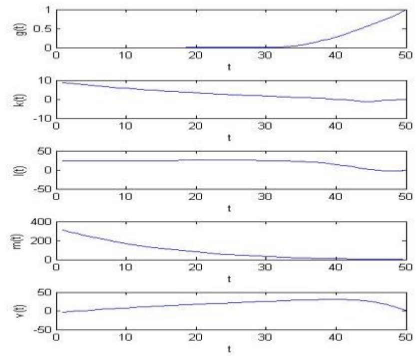

Figure 4: Plot of weight of bond (π_B) and weight of money account (π_B)

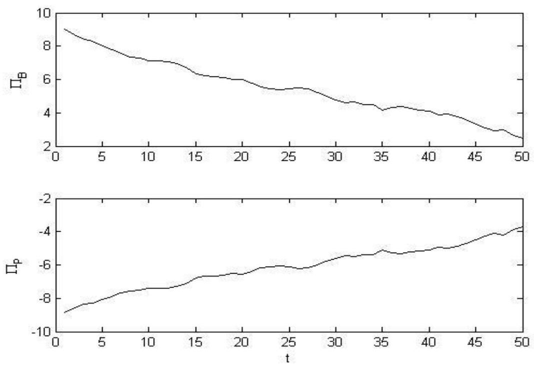

Figure 5: Plot of weight of bond (π_B) and weight of money account (π_B) for different write down rate ω = 0,1 (dotted-line), ω = 0,5 (dashed-line) and ω = 0,9(dot-dashed line).

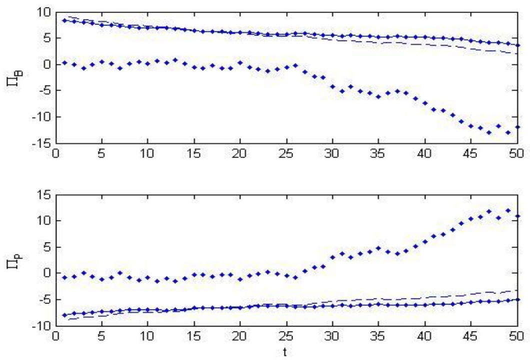

Figure 6: Plot of weight of bond (π_B) and weight of money account (π_B) for different utility coefficient
(γ = 0,1, γ = 0,5 and γ = 0,9)

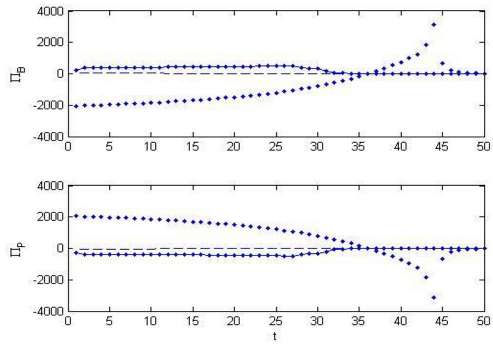

Figure (4) describes the trend of bond and money proportion. We see that both proportions are symmetric, which validate the model stated in Theorem 2. In figure (5) we try to test the sensitivity of weight of assets by changing the write down rate. This gives a meaning that investor can choose earlier which write down rate that they desire to have. Dotted-line, dashed-line, and dash-dotted line represent weight of assets under ω = 0.1, 0.3 and 0.5 respectively. It can be seen that for ω = 0, 1 the demand for bond is lower than other higher write down rates. Figure (6) describes the sensitivity of weight of assets when the utility functions are varied. Dotted-line, dashed-line, and dash-dotted line represent weight of assets under γ = 0.1, 0.3 and 0.5 respectively. It can be seen that dotted-line is lower than the other lines, and dashed-line has lower demand begin from half period of investment. This result can be interpreted that for more averse investor, they will took lower weight of bond instead of money account.

9. CONCLUSION

From literature review we can see that the study in dynamic portfolio by linking the market risks between rate of return and rate of inflation and credit risk in vasicek model into the asset, is still not done by other studies. The urgency to set this problem in Indonesia is based on the goal of monetary policy from Indonesian Bank Central, which is to control the inflation rate as the target value of monetary and also to have an exit from the investment when default occurred. Beside infla tion rate, the credit spread is appeared in this problem since that this component is very important to describe the credit risk. This is need to be considered since that the growth of corporate bond in Indonesia is well increased. Therefore the study of modelling the optimal portfolio under market and default risk is interesting to do, especially in Indonesia. The result is given in a closed loop model of wealth and also weight composition of assets, where it is depend on time function, market risk and credit spread rate. From the simulation and the sensitivity analysis of the finding model we can see that the utility and weight of assets are calibrated well following the behavior of exponential function and also has the same interpretation with what the investor will behave.

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APPENDIX

$A_1 = \zeta_\delta \sigma_\delta \bar{\lambda}_\delta + \zeta_I \sigma_I \bar{\lambda}_I$

$A_2 = \kappa_\delta \theta_\eta + \sigma_\delta \bar{\lambda}_\delta$

$A_3 = \kappa_I \theta_I + \sigma_I \bar{\lambda}_I$

$A_4 = (\zeta_\delta \sigma_\delta)^2 + 2\rho_{\delta I} \zeta_\delta \zeta_I \sigma_\delta \sigma_I + (\zeta_I \sigma_I)^2$

$A_5 = \frac{1}{2}\sigma_\delta^2$

$A_6 = \frac{1}{2}\sigma_I^2$

$A_7 = \sigma_\delta(\zeta_\delta \sigma_\delta + \rho_{\delta I} \zeta_I \sigma_I)$

$A_8 = \sigma_I(\zeta_I \sigma_I + \rho_{\delta I} \zeta_\delta \sigma_\delta)$

$A_9 = \rho_{\delta I} \sigma_\delta \sigma_I$

$C_1 = 2\left(\kappa_\delta + \frac{1-\gamma}{\gamma} . \frac{A_7}{A_4}\right)$

$C_2 = 2A_5$

$C_3 = -\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-A_7^2}{A_4}\right)$

$C_4 = -\gamma A_4$

$C_5 = \gamma A_4 \kappa_\delta - (1-\gamma)A_7$

$C_6 = -\gamma A_2 A_4 - (1-\gamma)A_1 A_7$

$C_7 = (1-\gamma)A_8$

$C_8 = -(1-\gamma)A_7^2 - 2\gamma A_4 A_5$

$C_9 = -(1-\gamma)A_7 A_8 - 2\gamma A_4 A_9$

$C_{10} = (1-\gamma)A_7$

$C_{11} = -2\kappa_\delta + 2\left(\frac{1-\gamma}{\gamma}\right) . \frac{A_8}{A_4}$

$C_{12} = 2A_6$

$C_{13} = \left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1+A_8^2}{A_4}\right)$

$C_{14} = -\gamma A_4$

$C_{15} = \gamma A_4 \kappa_I - (1-\gamma)A_8$

$C_{16} = -\gamma A_3 A_4 - (1-\gamma)A_1 A_8$

$C_{17} = -(1-\gamma)A_7$

$C_{18} = -(1-\gamma)A_8^2 - 2\gamma A_4 A_6$

$C_{19} = -(1-\gamma)A_7 A_8 - \gamma A_4 A_9$

$C_{20} = -(1-\gamma)A_1$

$C_{21} = -A_7 A_8 - \left(\frac{\gamma}{1-\gamma}\right) A_4 A_9$

$C_{22} = \frac{C_5}{C_4}$

$C_{23} = \frac{C_8}{C_4}$

$C_{24} = \frac{C_7}{C_4}$

$C_{25} = \frac{C_9}{C_4}$

$C_{26} = \frac{C_6}{C_4}$

$C_{27} = \frac{C_{10}}{C_4}$

$C_{28} = \frac{C_{15}}{C_{14}}$

$C_{29} = \frac{C_{18}}{C_{14}}$

$C_{30} = \frac{C_{19}}{C_{14}}$

$C_{31} = \frac{C_{19}}{C_{14}}$

$\bar{A}_1 = -(C_1 + 2C_2 C_3)$

$\bar{A}_2 = \frac{C_2 C_3 - A_1}{C_3}$

$\bar{B}_1 = -(C_{11} + 2C_{12} C_{13})$

$\bar{B}_2 = \frac{C_{12} C_{13} - A_1}{C_{13}}$

$\bar{B} = -(C_{10} + 2C_{11} C_{12})$